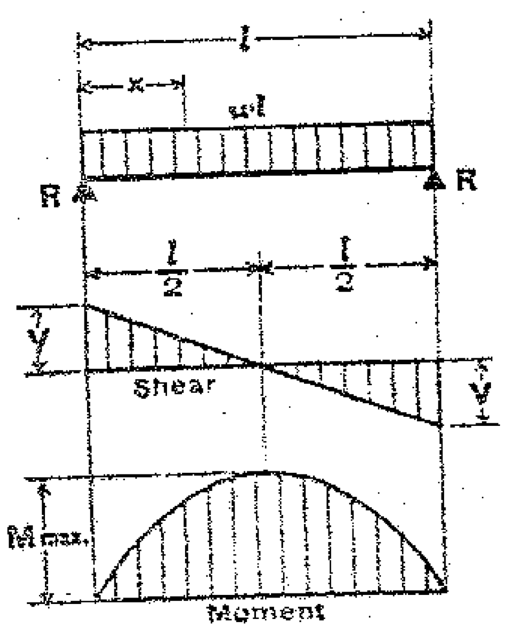
STRUCTURAL — BEAM FORMULAS - I

NOMENCLATURE

R = reaction — M = moment

V = shear — Δ = deflection

1. SIMPLE BEAM—UNIFORMLY DISTRIBUTED LOAD



Equivalent Tabular Load . . . . $= wl$

$R = V$ . . . . $= \frac{wl}{2}$

V_x . . . . $= w\left(\frac{l}{2} - x\right)$

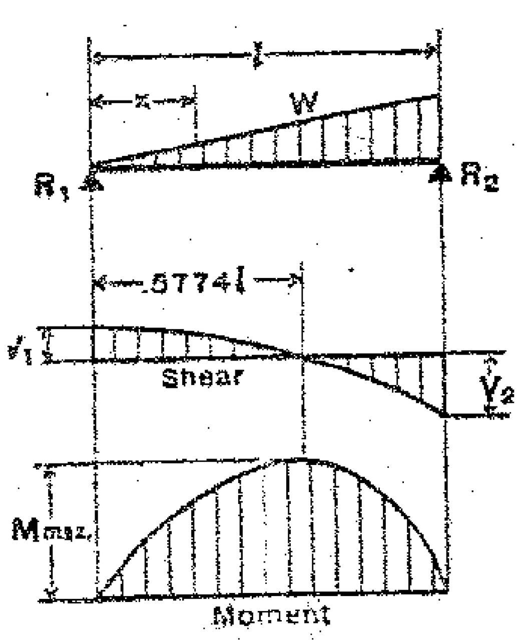
M max. (at center) . . . . . $= \frac{wl^2}{8}$

M_x . . . . $= \frac{wx}{2}(l - x)$

Δmax. (at center) . . . . . $= \frac{5\,wl^4}{384\,EI}$

Δx . . . . $= \frac{wx}{24EI}(l^3 - 2lx^2 + x^3)$

2. SIMPLE BEAM—LOAD INCREASING UNIFORMLY TO ONE END



Equivalent Tabular Load . . . . $= \frac{16W}{9\sqrt{3}} = 1.0264W$

$R_1 = V_1$ . . . . $= \frac{W}{3}$

$R_2 = V_2$ max. . . . . $= \frac{2W}{3}$

V_x . . . . $= \frac{W}{3} - \frac{Wx^2}{l^2}$

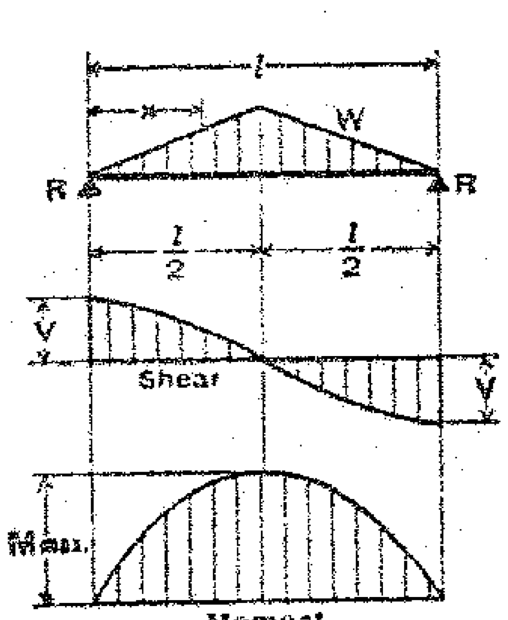
M max. $\left(\text{at } x = \frac{l}{\sqrt{3}} = .5774l\right)$ . . . $= \frac{2Wl}{9\sqrt{3}} = .1283\ Wl$

M_x . . . . $= \frac{Wx}{3l^2}(l^2 - x^2)$

Δmax. $\left(\text{at } x = l\sqrt{1 - \sqrt{\frac{8}{15}}} = .5193l\right)$ $= .01304\,\frac{Wl^3}{EI}$

Δx . . . . $= \frac{Wx}{180EI\,l^2}(3x^4 - 10l^2x^2 + 7l^4)$

3. SIMPLE BEAM—LOAD INCREASING UNIFORMLY TO CENTER



Equivalent Tabular Load . . . . $= \frac{4W}{3}$

$R = V$ . . . . $= \frac{W}{2}$

V_x $\left(\text{when } x < \frac{l}{2}\right)$ . . . . . $= \frac{W}{2l^2}(l^2 - 4x^2)$

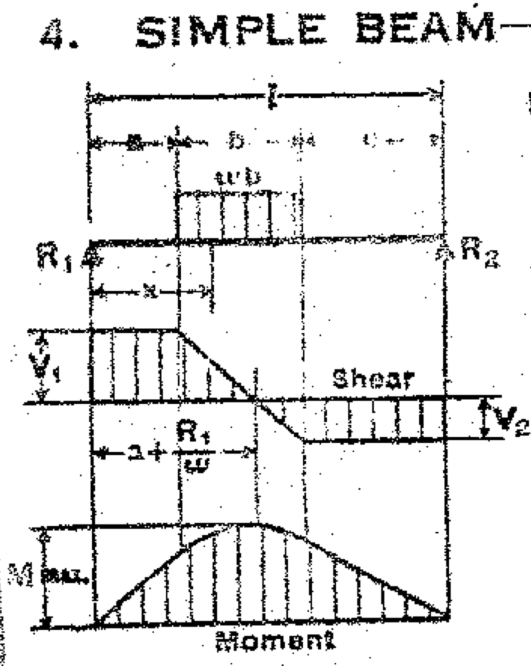
M max. (at center) . . . . . $= \frac{Wl}{6}$

M_x $\left(\text{when } x < \frac{l}{2}\right)$ . . . . . $= Wx\left(\frac{1}{2} - \frac{2x^2}{3l^2}\right)$

Δmax. (at center) . . . . . $= \frac{Wl^3}{60EI}$

Δx . . . . $= \frac{Wx}{480\,EI\,l^2}(5l^2 - 4x^2)^2$

4. SIMPLE BEAM—UNIFORM LOAD PARTIALLY DISTRIBUTED



$R_1 = V_1$ (max. when $a < c$) . . . $= \frac{wb}{2l}(2c + b)$

$R_2 = V_2$ (max. when $a > c$) . . . $= \frac{wb}{2l}(2a + b)$

V_x (when $x > a$ and $< (a + b)$) . $= R_1 - w(x - a)$

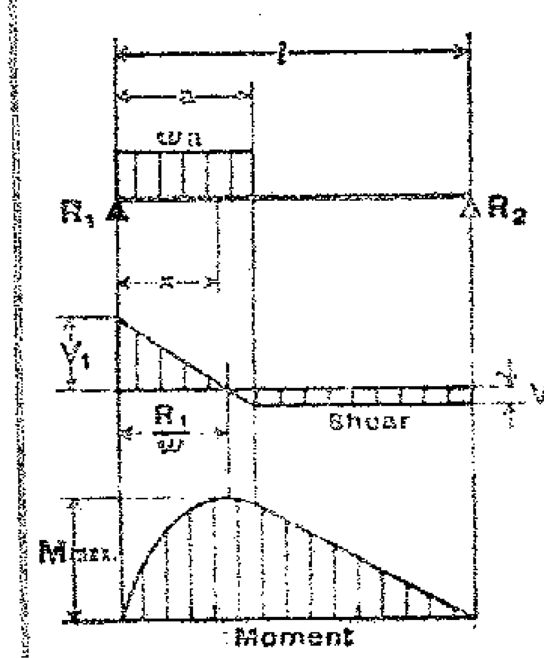
M max. $\left(\text{at } x = a + \frac{R_1}{w}\right)$ . . . . $= R_1\left(a + \frac{R_1}{2w}\right)$

M_x (when $x < a$) . . . . . . $= R_1x$

M_x (when $x > a$ and $< (a + b)$) . $= R_1x - \frac{w}{2}(x - a)^2$

M_x (when $x > (a + b)$) . . . . $= R_2(l - x)$

5. SIMPLE BEAM—UNIFORM LOAD PARTIALLY DISTRIBUTED AT ONE END



$R_1 = V_1$ max. . . . . $= \frac{wa}{2l}(2l - a)$

$R_2 = V_2$ . . . . $= \frac{wa^2}{2l}$

V (when $x < a$) . . . . . . $= R_1 - wx$

M max. $\left(\text{at } x = \frac{R_1}{w}\right)$ . . . . . . $= \frac{R_1^2}{2w}$

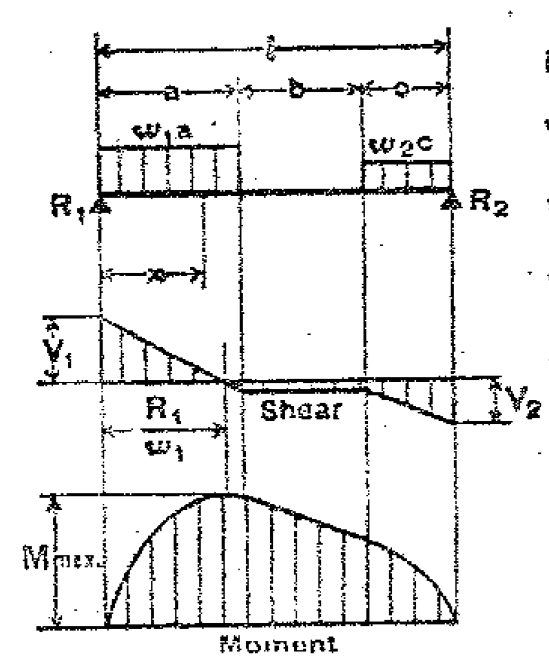
M_x (when $x < a$) . . . . . . $= R_1x - \frac{wx^2}{2}$

M_x (when $x > a$) . . . . . . $= R_2(l - x)$

Δx (when $x < a$) . . . . . . $= \frac{wx}{24EIl}\left(a^2(2l-a)^2 - 2ax^2(2l-a) + lx^3\right)$

Δx (when $x > a$) . . . . . . $= \frac{wa^2(l - x)}{24EIl}(4xl - 2x^2 - a^2)$

6. SIMPLE BEAM—UNIFORM LOAD PARTIALLY DISTRIBUTED AT EACH END



$R_1 = V_1$ . . . . $= \frac{w_1a(2l - a) + w_2c^2}{2l}$

$R_2 = V_2$ . . . . $= \frac{w_2c(2l - c) + w_1a^2}{2l}$

V_x (when $x < a$) . . . . . . $= R_1 - w_1x$

V_x (when $x > a$ and $< (a + b)$) . $= R_1 - R_2$

V_x (when $x > (a + b)$) . . . . $= R_2 - w_2(l - x)$

M max. $\left(\text{at } x = \frac{R_1}{w_1} \text{ when } R_1 < w_1a\right)$. $= \frac{R_1^2}{2w_1}$

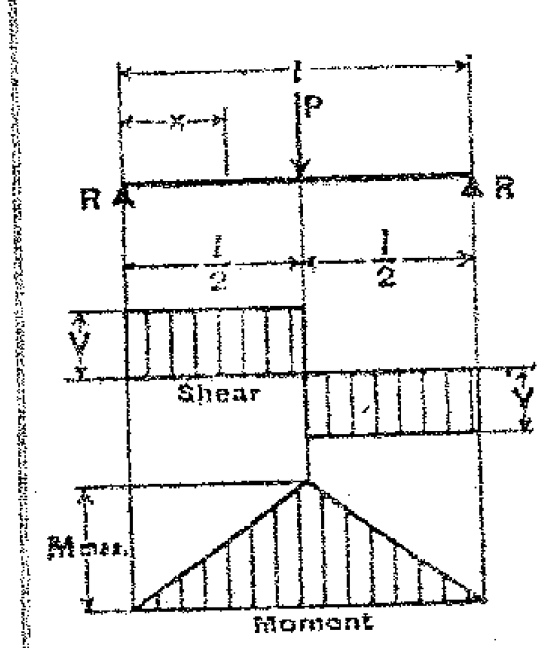
M max. $\left(\text{at } x = l - \frac{R_2}{w_2} \text{ when } R_2 < w_2c\right)$ $= \frac{R_2^2}{2w_2}$

M_x (when $x < a$) . . . . . . $= R_1x - \frac{w_1x^2}{2}$

M_x (when $x > a$ and $< (a + b)$) . $= R_1x - \frac{w_1a}{2}(2x - a)$

M_x (when $x > (a + b)$) . . . . $= R_2(l - x) - \frac{w_2(l - x)^2}{2}$

7. SIMPLE BEAM—CONCENTRATED LOAD AT CENTER



Equivalent Tabular Load . . . . . . $= 2P$

$R = V$ . . . . $= \frac{P}{2}$

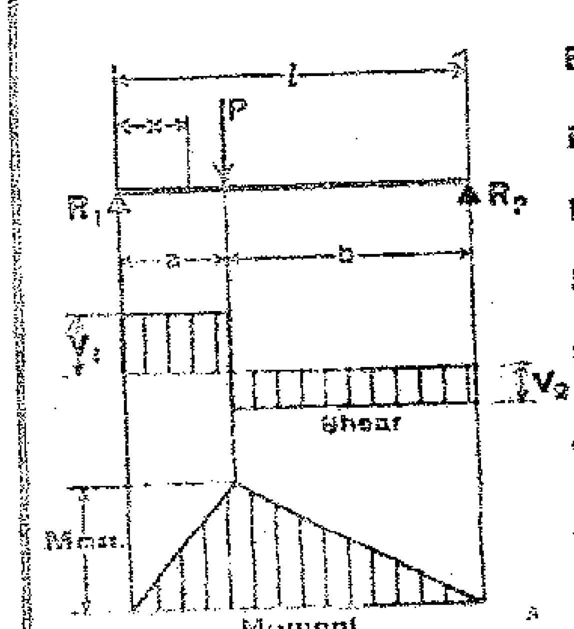
M max. (at point of load) . . . . . $= \frac{Pl}{4}$

M_x $\left(\text{when } x < \frac{l}{2}\right)$ . . . . . $= \frac{Px}{2}$

Δmax. (at point of load) . . . . . $= \frac{Pl^3}{48EI}$

Δx $\left(\text{when } x < \frac{l}{2}\right)$ . . . . . $= \frac{Px}{48EI}(3l^2 - 4x^2)$

8. SIMPLE BEAM—CONCENTRATED LOAD AT ANY POINT



Equivalent Tabular Load . . . . . . $= \frac{8\,Pab}{l^2}$

$R_1 = V_1$ (max. when $a < b$) . . . . $= \frac{Pb}{l}$

$R_2 = V_2$ (max. when $a > b$) . . . . $= \frac{Pa}{l}$

M max. (at point of load) . . . . $= \frac{Pab}{l}$

M_x (when $x < a$) . . . . $= \frac{Pbx}{l}$

Δmax. $\left(\text{at } x = \sqrt{\frac{a(a + 2b)}{3}} \text{ when } a > b\right)$ $= \frac{Pab(a + 2b)\sqrt{3a(a + 2b)}}{27\,EI\,l}$

Δa (at point of load) . . . . . $= \frac{Pa^2b^2}{3EIl}$

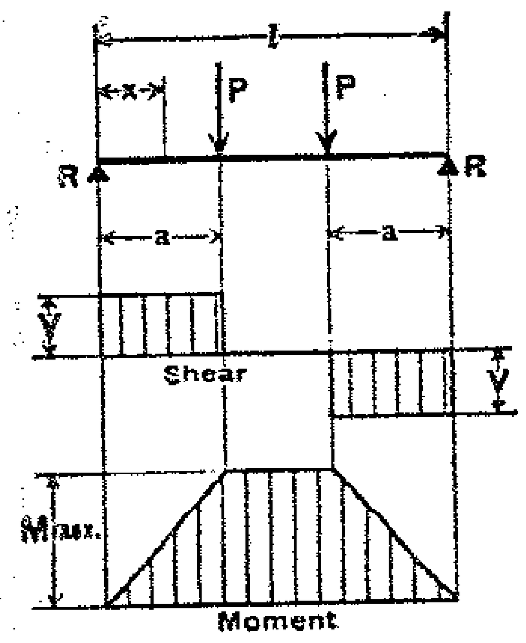
Δx (when $x < a$) . . . . . $= \frac{Pbx}{6EIl}(l^2 - b^2 - x^2)$

From A.I.S.C. Steel Construction Manual - 1950.

STRUCTURAL — BEAM FORMULAS-2

For nomenclature see page 8-01

9. SIMPLE BEAM—TWO EQUAL CONCENTRATED LOADS SYMMETRICALLY PLACED



Equivalent Tabular Load . . . . . . $= \frac{8Pa}{l}$

$R = V$ . . . . $= P$

M max. (between loads) . . . . . . $= Pa$

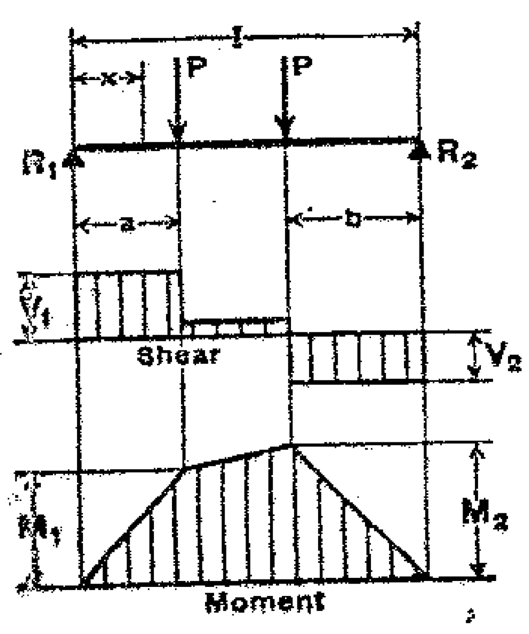
M_x (when $x < a$) . . . . $= Px$

Δmax. (at center) . . . . . . $= \frac{Pa}{24EI}(3l^2 - 4a^2)$

Δx (when $x < a$) . . . . . . $= \frac{Px}{6EI}(3la - 3a^2 - x^2)$

Δx (when $x > a$ and $< (l-a)$) . . $= \frac{Pa}{6EI}(3lx - 3x^2 - a^2)$

10. SIMPLE BEAM—TWO EQUAL CONCENTRATED LOADS UNSYMMETRICALLY PLACED



$R_1 = V_1$ (max. when $a < b$) . . . . . $= \frac{P}{l}(l - a + b)$

$R_2 = V_2$ (max. when $a > b$) . . . . . $= \frac{P}{l}(l - b + a)$

V_x (when $x > a$ and $< (l-b)$) . . $= \frac{P}{l}(b - a)$

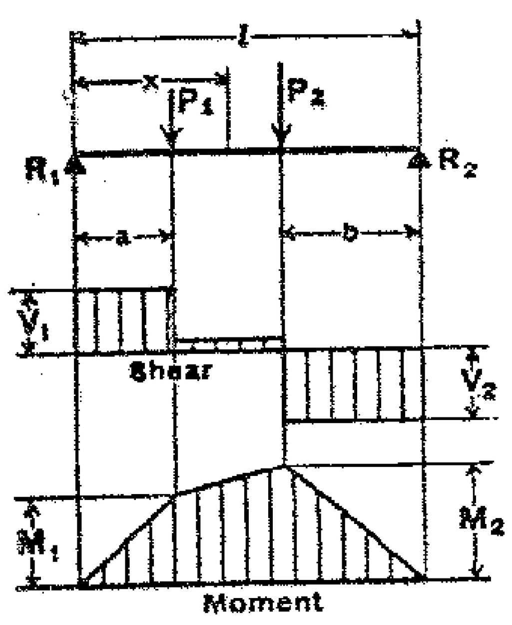
M_1 (max. when $a > b$) . . . . . $= R_1 a$

M_2 (max. when $a < b$) . . . . . $= R_2 b$

M_x (when $x < a$) . . . . $= R_1 x$

M_x (when $x > a$ and $< (l-b)$) . . $= R_1 x - P(x - a)$

11. SIMPLE BEAM—TWO UNEQUAL CONCENTRATED LOADS UNSYMMETRICALLY PLACED



$R_1 = V_1$ . . . . $= \frac{P_1(l-a) + P_2 b}{l}$

$R_2 = V_2$ . . . . $= \frac{P_1 a + P_2(l-b)}{l}$

V_x (when $x > a$ and $< (l-b)$) . . $= R_1 - P_1$

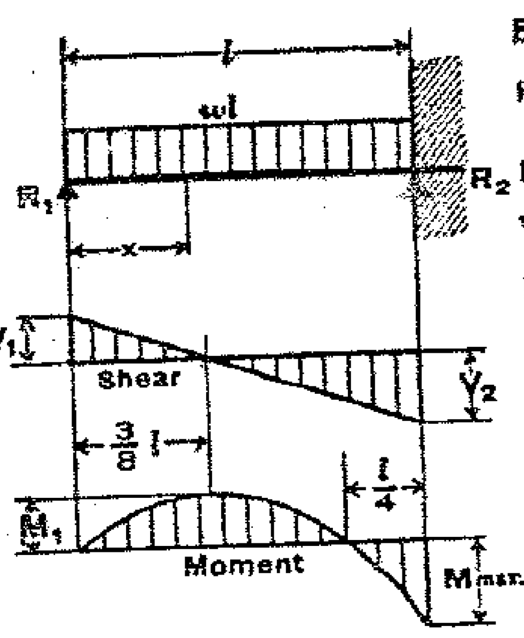
M_1 (max. when $R_1 < P_1$) . . . . $= R_1 a$

M_2 (max. when $R_2 < P_2$) . . . . $= R_2 b$

M_x (when $x < a$) . . . . $= R_1 x$

M_x (when $x > a$ and $< (l-b)$) . . $= R_1 x - P_1(x - a)$

12. BEAM FIXED AT ONE END, SUPPORTED AT OTHER—UNIFORMLY DISTRIBUTED LOAD



Equivalent Tabular Load . . . . . . $= wl$

$R_1 = V_1$ . . . . $= \frac{3wl}{8}$

$R_2 = V_2$ max. . . . . $= \frac{5wl}{8}$

V_x . . . . $= R_1 - wx$

M max. . . . . $= \frac{wl^2}{8}$

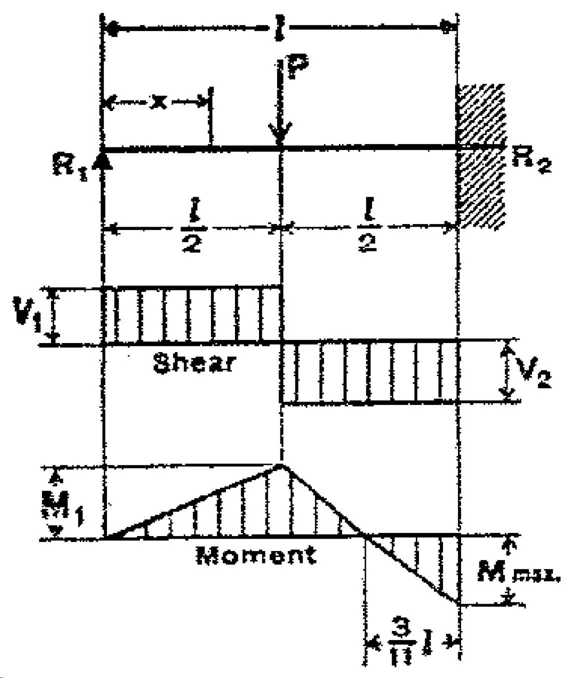
M_1 (at $x = \frac{3}{8}l$) . . . . $= \frac{9}{128}wl^2$

M_x . . . . $= R_1 x - \frac{wx^2}{2}$

Δmax. (at $x = \frac{l}{16}(1 + \sqrt{33}) = .4215l$) . $= \frac{wl^4}{185EI}$

Δx . . . . $= \frac{wx}{48EI}(l^3 - 3lx^2 + 2x^3)$

13. BEAM FIXED AT ONE END, SUPPORTED AT OTHER—CONCENTRATED LOAD AT CENTER



Equivalent Tabular Load . . . . . $= \frac{3P}{2}$

$R_1 = V_1$ . . . . $= \frac{5P}{16}$

$R_2 = V_2$ max. . . . . $= \frac{11P}{16}$

M max. (at fixed end) . . . . . . $= \frac{3Pl}{16}$

M_1 (at point of load) . . . . . $= \frac{5Pl}{32}$

M_x (when $x < \frac{l}{2}$) . . . . . . $= \frac{5Px}{16}$

M_x (when $x > \frac{l}{2}$) . . . . . . $= P\left(\frac{l}{2} - \frac{11x}{.16}\right)$

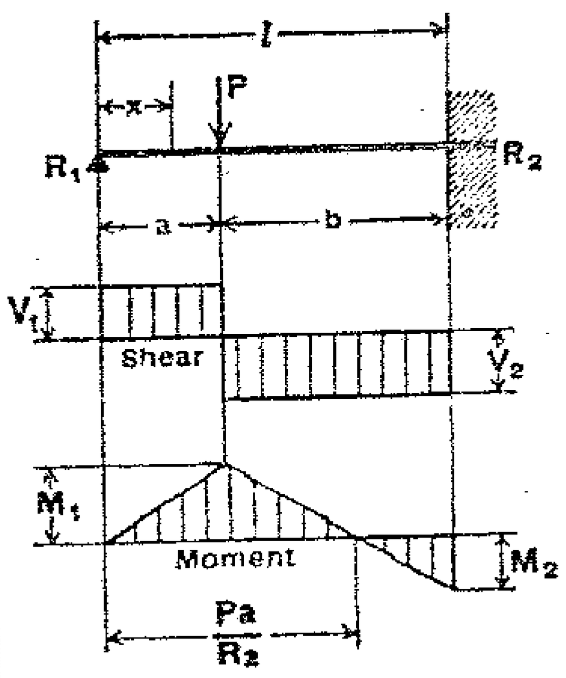
Δmax. (at $x = l\sqrt{\frac{1}{5}} = .4472l$) . . $= \frac{Pl^3}{48EI\sqrt{5}} = .009317\frac{Pl^3}{EI}$

Δx (at point of load) . . . . . $= \frac{7Pl3}{768EI}$

Δx (when $x < \frac{l}{2}$) . . . . . . $= \frac{Px}{96EI}(3l^2 - 5x^2)$

Δx (when $x > \frac{l}{2}$) . . . . . . $= \frac{P}{96EI}(x - l)^2(11x - 2l)$

14. BEAM FIXED AT ONE END, SUPPORTED AT OTHER—CONCENTRATED LOAD AT ANY POINT



$R_1 = V_1$ . . . . $= \frac{Pb^2}{2l^3}(a + 2l)$

$R_2 = V_2$ . . . . $= \frac{Pa}{2l^3}(3l^2 - a^2)$

M_1 (at point of load) . . . . . $= R_1 a$

M_2 (at fixed end) . . . . . . $= \frac{Pab}{2l^2}(a + l)$

M_x (when $x < a$) . . . . . . $= R_1 x$

M_x (when $x > a$) . . . . . . $= R_1 x - P(x - a)$

Δmax. (when $a < .414l$ at $x = l\frac{l^2 + a^2}{3l^2 - a^2}$) $= \frac{Pa}{3EI}\frac{(l^2 - a^2)^3}{(3l^2 - a^2)^2}$

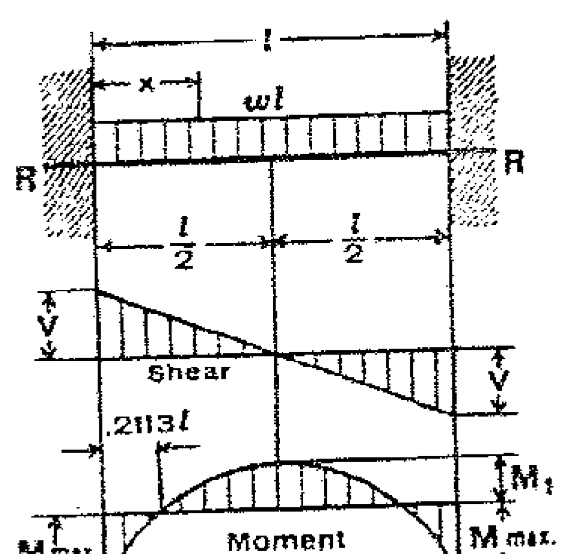
Δmax. (when $a > .414l$ at $x = l\sqrt{\frac{a}{2l + a}}$) $= \frac{Pab^2}{6EI}\sqrt{\frac{a}{2l + a}}$

Δa (at point of load) . . . . . $= \frac{Pa^2b^3}{12EIl^3}(3l + a)$

Δx (when $x < a$) . . . . . . $= \frac{Pb^2x}{12EIl^3}(3al^2 - 2lx^2 - ax^2)$

Δx (when $x > a$) . . . . . . $= \frac{Pa}{12EIl^3}(l - x)^2(3l^2x - a^2x - 2a^2l)$

15. BEAM FIXED AT BOTH ENDS—UNIFORMLY DISTRIBUTED LOADS



Equivalent Tabular Load . . . . . $= \frac{2wl}{3}$

$R = V$ . . . . $= \frac{wl}{2}$

V_x . . . . $= w\left(\frac{l}{2} - x\right)$

M max. (at ends) . . . . $= \frac{wl^2}{12}$

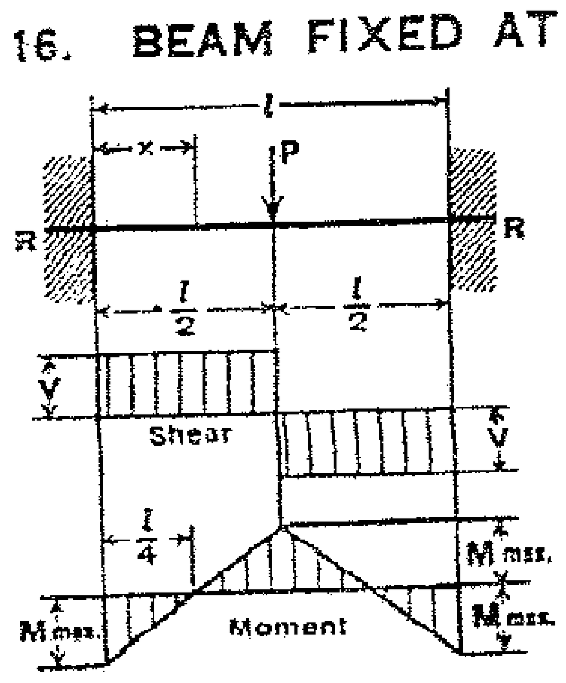
M_1 (at center) . . . . $= \frac{wl^2}{24}$

M_x . . . . $= \frac{w}{12}(6lx - l^2 - 6x^2)$

Δmax. (at center) . . . . . . $= \frac{wl^4}{384EI}$

Δx . . . . $= \frac{wx^2}{24EI}(l - x)^2$

16. BEAM FIXED AT BOTH ENDS—CONCENTRATED LOAD AT CENTER



Equivalent Tabular Load . . . . . $= P$

$R = V$ . . . . $= \frac{P}{2}$

M max. (at center and ends) . . . $= \frac{Pl}{8}$

M_x (when $x < \frac{l}{2}$) . . . . . . $= \frac{P}{8}(4x - l)$

Δmax. (at center) . . . . . . $= \frac{Pl^3}{192EI}$

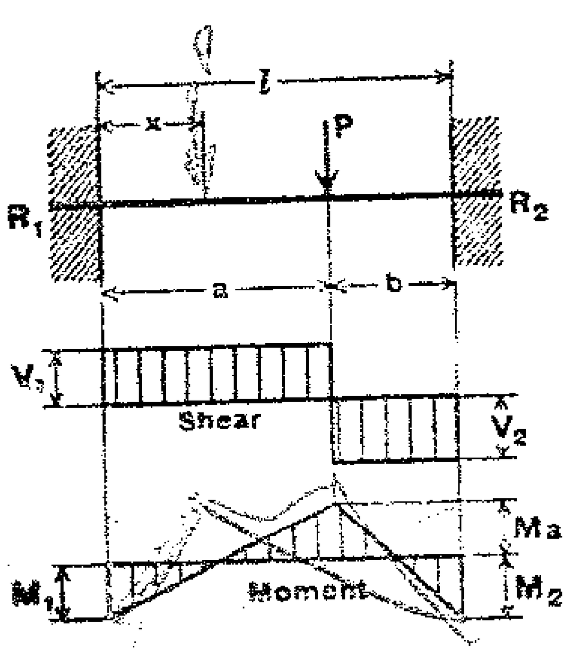
Δx . . . . $= \frac{Px^2}{48EI}(3l - 4x)$

From A.I.S.C. Steel Construction Manual - 1950.

STRUCTURAL — BEAM FORMULAS—3

For nomenclature see page 8-01

17. BEAM FIXED AT BOTH ENDS—CONCENTRATED LOAD AT ANY POINT



$R_1 = V_1$ (max. when a < b) . . . . $= \frac{Pb^2}{l^3}(3a + b)$

$R_2 = V_2$ (max. when a > b) . . . . $= \frac{Pa^2}{l^3}(a + 3b)$

M_1 (max. when a < b) . . . . $= \frac{Pab^2}{l^2}$

M_2 (max. when a > b) . . . . $= \frac{Pa^2b}{l^2}$

M_a (at point of load) . . . . $= \frac{2Pa^2b^2}{l^3}$

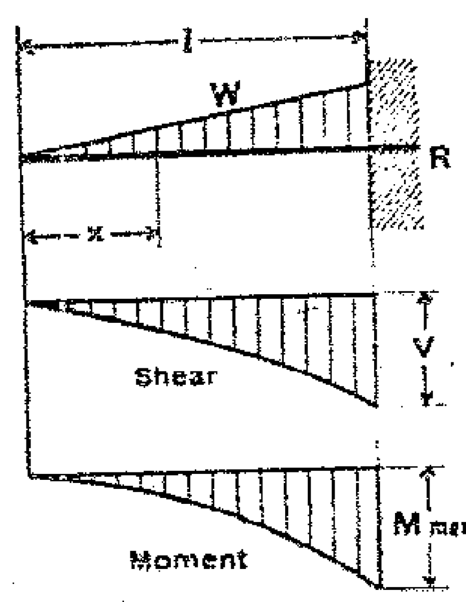
M_x (when x < a) . . . . $= R_1x - \frac{Pab^2}{l^2}$

Δmax. (when a > b at $x = \frac{2al}{3a+b}$) . $= \frac{2Pa^3b^2}{3EI\,(3a+b)^2}$

Δa (at point of load) . . . . $= \frac{Pa^3b^3}{3EIl^3}$

Δx (when x < a) . . . . $= \frac{Pb^2x^2}{6EIl^3}(3al - 3ax - bx)$

18. CANTILEVER BEAM—LOAD INCREASING UNIFORMLY TO FIXED END



Equivalent Tabular Load . . . . $= \frac{8}{3}W$

R = V . . . . $= W$

V_x . . . . $= W\frac{x^2}{l^2}$

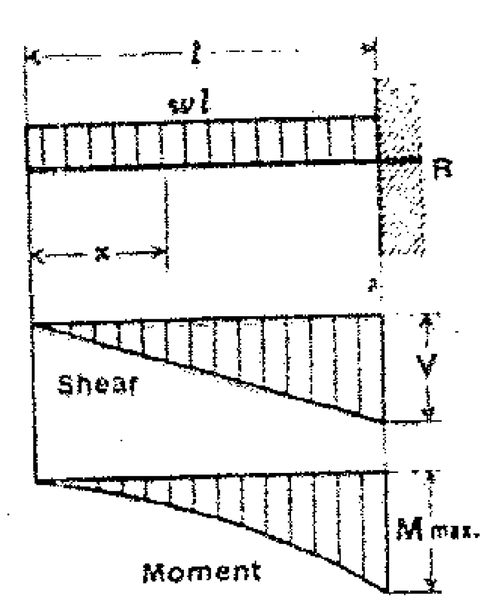
M max. (at fixed end) . . . . $= \frac{Wl}{3}$

M_x . . . . $= \frac{Wx^3}{3l^2}$

Δmax. (at free end) . . . . $= \frac{Wl^3}{15EI}$

Δx . . . . $= \frac{W}{60EIl^2}(x^5 - 5l^4x + 4l^5)$

19. CANTILEVER BEAM—UNIFORMLY DISTRIBUTED LOAD



Equivalent Tabular Load . . . . $= 4wl$

R = V . . . . $= wl$

V_x . . . . $= wx$

M max. (at fixed end) . . . . $= \frac{wl^2}{2}$

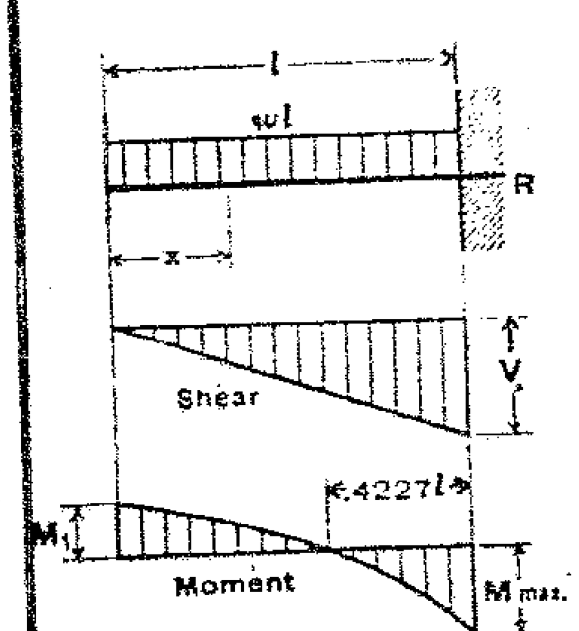
M_x . . . . $= \frac{wx^2}{2}$

Δmax. (at free end) . . . . $= \frac{wl^4}{8EI}$

Δx . . . . $= \frac{w}{24EI}(x^4 - 4l^3x + 3l^4)$

20. BEAM FIXED AT ONE END, FREE BUT GUIDED AT OTHER—UNIFORMLY DISTRIBUTED LOAD

The deflection at the guided end is assumed to be in a vertical plane.



Equivalent Tabular Load . . . . $= \frac{8}{3}wl$

R = V . . . . $= wl$

V_x . . . . $= wx$

M max. (at fixed end) . . . . $= \frac{wl^2}{3}$

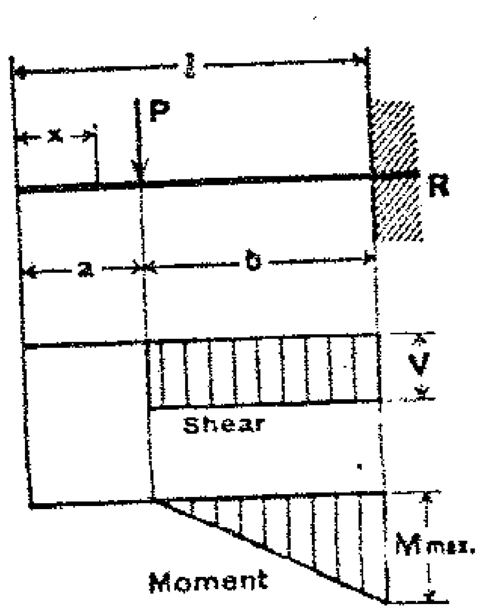
M_1 (at guided end) . . . . $= \frac{wl^2}{6}$

M_x . . . . $= \frac{w}{6}(l^2 - 3x^2)$

Δmax. (at guided end) . . . . $= \frac{wl^4}{24EI}$

Δx . . . . $= \frac{w(l^2 - x^2)^2}{24EI}$

21. CANTILEVER BEAM—CONCENTRATED LOAD AT ANY POINT



Equivalent Tabular Load . . . . $= \frac{8Pb}{l}$

R = V (when x < a) . . . . $= P$

M max. (at fixed end) . . . . $= Pb$

M_x (when x > a) . . . . $= P(x - a)$

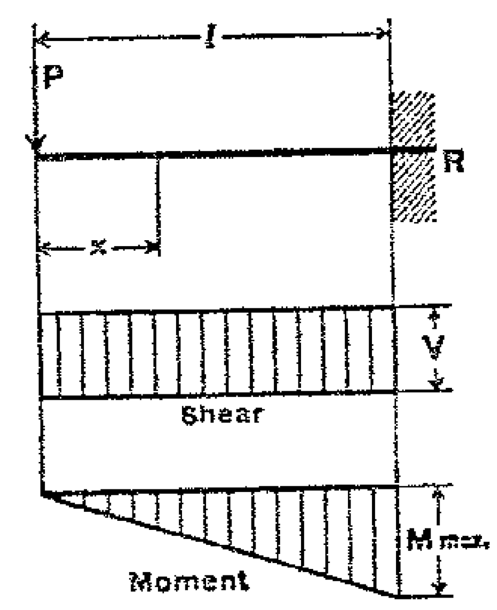
Δmax. (at free end) . . . . $= \frac{Pb^2}{6EI}(3l - b)$

Δa (at point of load) . . . . $= \frac{Pb^3}{3EI}$

Δx (when x < a) . . . . $= \frac{Pb^2}{6EI}(3l - 3x - b)$

Δx (when x > a) . . . . $= \frac{P(l - x)^2}{6EI}(3b - l + x)$

22. CANTILEVER BEAM—CONCENTRATED LOAD AT FREE END



Equivalent Tabular Load . . . . $= 8P$

R = V . . . . $= P$

M max. (at fixed end) . . . . $= Pl$

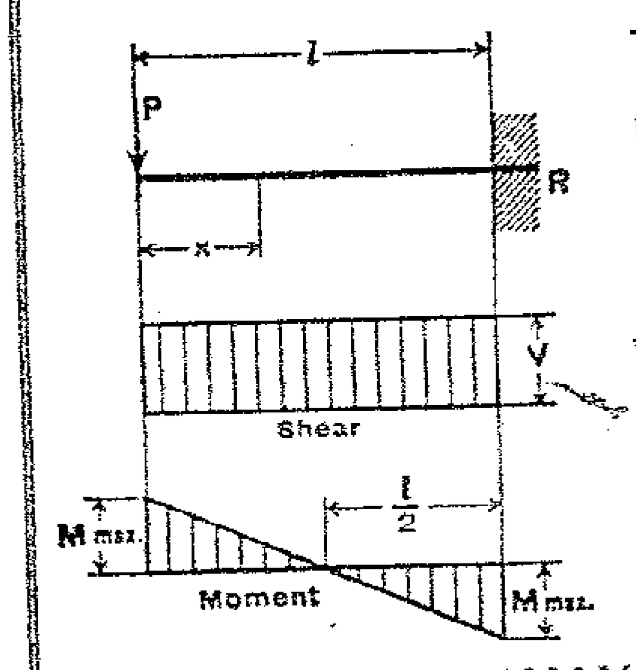
M_x . . . . $= Px$

Δmax. (at free end) . . . . $= \frac{Pl^3}{3EI}$

Δx . . . . $= \frac{P}{6EI}(2l^3 - 3l^2x + x^3)$

23. BEAM FIXED AT ONE END, FREE BUT GUIDED AT OTHER—CONCENTRATED LOAD AT GUIDED END

The deflection at the guided end is assumed to be in a vertical plane.



Equivalent Tabular Load . . . . $= 4P$

R = V . . . . $= P$

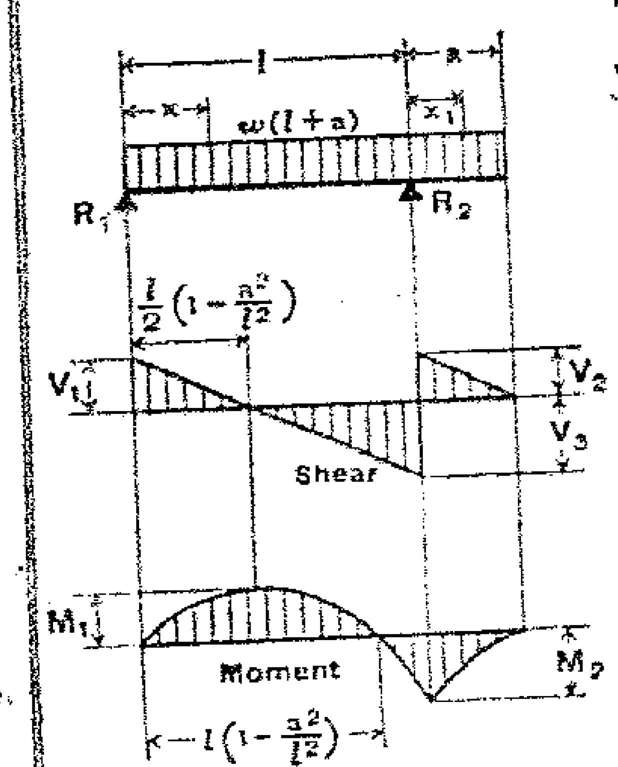
M max. (at both ends) . . . . $= \frac{Pl}{2}$

M_x . . . . $= P\left(\frac{l}{2} - x\right)$

Δmax. (at guided end) . . . . $= \frac{Pl^3}{12EI}$

Δx . . . . $= \frac{P(l - x)^2}{12EI}(l + 2x)$

24. BEAM OVERHANGING ONE SUPPORT—UNIFORMLY DISTRIBUTED LOAD



$R_1 = V_1$ . . . . $= \frac{w}{2l}(l^2 - a^2)$

$R_2 = V_2 + V_3$ . . . . $= \frac{w}{2l}(l + a)^2$

V_2 . . . . $= wa$

V_3 . . . . $= \frac{w}{2l}(l^2 + a^2)$

V_x (between supports) . . . . $= R_1 - wx$

V_{x_1} (for overhang) . . . . $= w(a - x_1)$

M_1 (at $x = \frac{l}{2}\left[1 - \frac{a^2}{l^2}\right]$) . . . . $= \frac{w}{8l^2}(l + a)^2(l - a)^2$

M_2 (at R_2) . . . . $= \frac{wa^2}{2}$

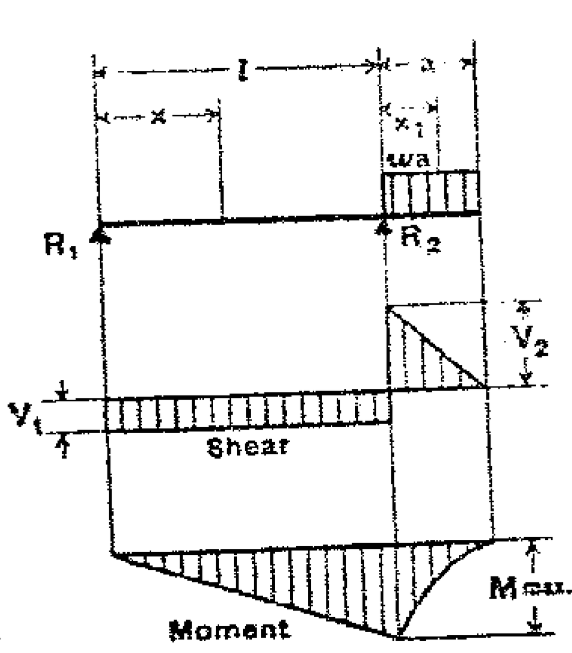
M_x (between supports) . . . . $= \frac{wx}{2l}(l^2 - a^2 - xl)$

M_{x_1} (for overhang) . . . . $= \frac{w}{2}(a - x_1)^2$

Δx (between supports) . . . . $= \frac{wx}{24EIl}(l^4 - 2l^2x^2 + lx^3 - 2a^2l^2 + 2a^2x^2)$

Δx_1 (for overhang) . . . . $= \frac{wx_1}{24EI}(4a^2l - l^3 + 6a^2x_1 - 4ax_1^2 + x_1^3)$

25. BEAM OVERHANGING ONE SUPPORT—UNIFORMLY DISTRIBUTED LOAD ON OVERHANG



$R_1 = V_1$ . . . . $= \frac{wa^2}{2l}$

$R_2 = V_1 + V_2$ . . . . $= \frac{wa}{2l}(2l + a)$

V_2 . . . . $= wa$

V_{x_1} (for overhang) . . . . $= w(a - x_1)$

M max. (at R_2) . . . . $= \frac{wa^2}{2}$

M_x (between supports) . . . . $= \frac{wa^2x}{2l}$

M_{x_1} (for overhang) . . . . $= \frac{w}{2}(a - x_1)^2$

Δmax. (between supports at $x = \frac{l}{\sqrt{3}}$) $= \frac{wa^2l^2}{18\sqrt{3}EI} = .03208\frac{wa^2l^2}{EI}$

Δmax. (for overhang at $x_1 = a$) . $= \frac{wa^3}{24EI}(4l + 3a)$

Δx (between supports) . . . . $= \frac{wa^2x}{12EIl}(l^2 - x^2)$

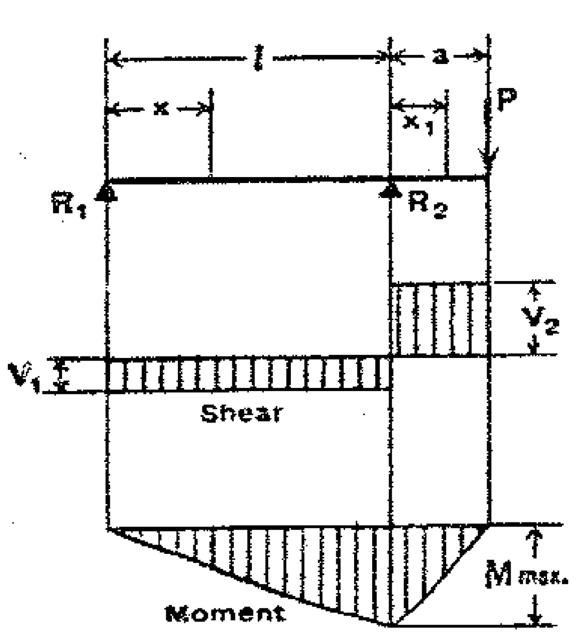
Δx_1 (for overhang) . . . . $= \frac{wx_1}{24EI}(4a^2l + 6a^2x_1 - 4ax_1^2 + x_1^3)$

From A.I.S.C. Steel Construction Manual - 1950.

STRUCTURAL — BEAM FORMULAS — 4

For nomenclature see page 8-01

26. BEAM OVERHANGING ONE SUPPORT—CONCENTRATED LOAD AT END OF OVERHANG



$R_1 = V_1$ . . . . . . $= \frac{Pa}{l}$

$R_2 = V_1 + V_2$ . . . . . . $= \frac{P}{l}(l+a)$

V_2 . . . . . . $= P$

M max. (at R_2) . . . . . . $= Pa$

M_x (between supports) . . . $= \frac{Pax}{l}$

M_{x_1} (for overhang) . . . $= P(a - x_1)$

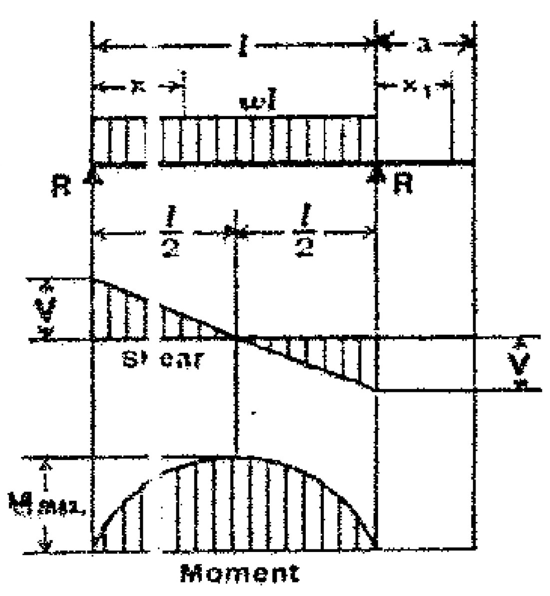
$\Delta max.$ (between supports at $x = \frac{l}{\sqrt{3}}$) $= \frac{Pal^2}{9\sqrt{3}EI} = .06415\frac{Pal^2}{EI}$

$\Delta max.$ (for overhang at $x_1 = a$) . $= \frac{Pa^2}{3EI}(l+a)$

Δx (between supports) . . . $= \frac{Pax}{6EIl}(l^2 - x^2)$

Δx_1 (for overhang) . . . $= \frac{Px_1}{6EI}(2al + 3ax_1 - x_1^2)$

27. BEAM OVERHANGING ONE SUPPORT—UNIFORMLY DISTRIBUTED LOAD BETWEEN SUPPORTS



Equivalent Tabular Load . . . . $= wl$

$R = V$ . . . . . . $= \frac{wl}{2}$

V_x . . . . . . $= w\left(\frac{l}{2} - x\right)$

M max. (at center) . . . . . $= \frac{wl^2}{8}$

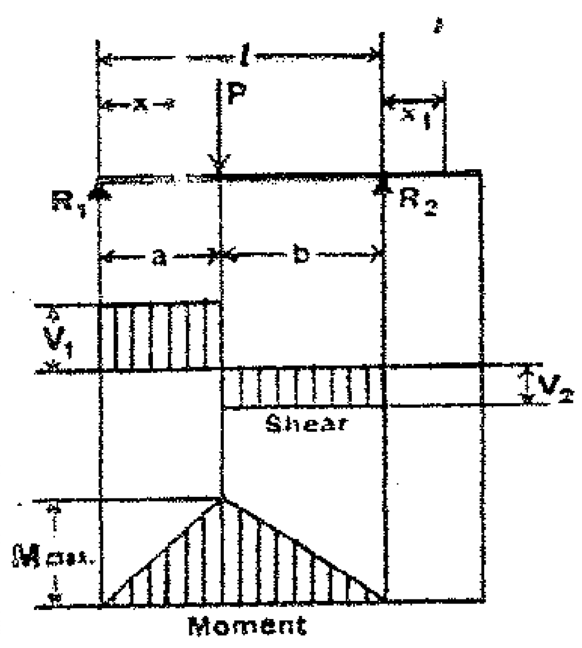
M_x . . . . . . $= \frac{wx}{2}(l - x)$

$\Delta max.$ (at center) . . . . $= \frac{5wl^4}{384EI}$

Δx . . . . . . $= \frac{wx}{24EI}(l^3 - 2lx^2 + x^3)$

Δx_1 . . . . . . $= \frac{wl^3x_1}{24EI}$

28. BEAM OVERHANGING ONE SUPPORT—CONCENTRATED LOAD AT ANY POINT BETWEEN SUPPORTS



Equivalent Tabular Load . . . $= \frac{8Pab}{l^2}$

$R_1 = V_1$ (max. when $a < b$) . . . $= \frac{Pb}{l}$

$R_2 = V_2$ (max. when $a > b$) . . . $= \frac{Pa}{l}$

M max. (at point of load) . . . $= \frac{Pab}{l}$

M_x (when $x < a$) . . . $= \frac{Pbx}{l}$

$\Delta max.$ (at $x = \sqrt{\frac{a(a+2b)}{3}}$ when $a > b$) $= \frac{Pab(a+2b)\sqrt{3a(a+2b)}}{27EIl}$

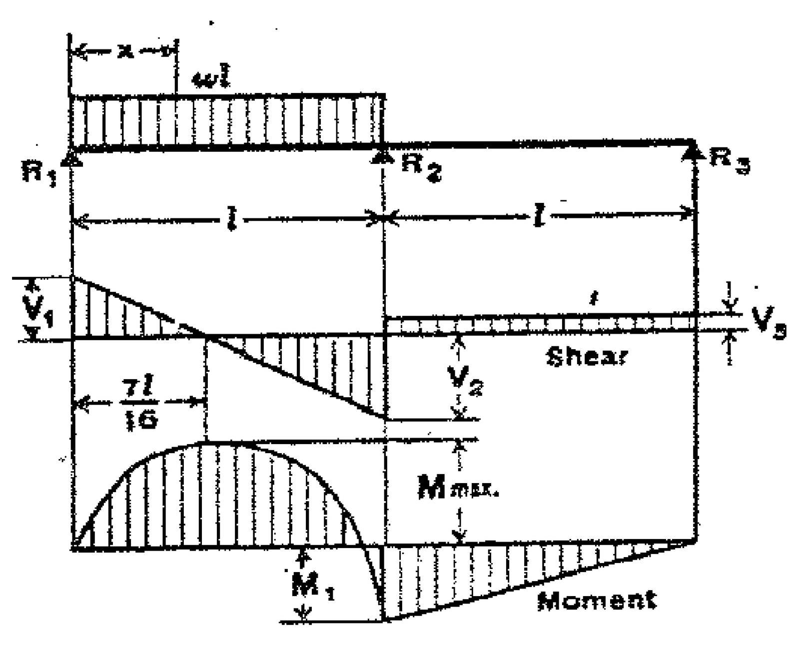
Δa (at point of load) . . . $= \frac{Pa^2b^2}{3EIl}$

Δx (when $x < a$) . . . $= \frac{Pbx}{6EIl}(l^2 - b^2 - x^2)$

Δx (when $x > a$) . . . $= \frac{Pa(l-x)}{6EIl}(2lx - x^2 - a^2)$

Δx_1 . . . . . . $= \frac{Pabx_1}{6EIl}(l+a)$

29. CONTINUOUS BEAM—TWO EQUAL SPANS—UNIFORM LOAD ON ONE SPAN



Equivalent Tabular Load . . $= \frac{49}{64}wl$

$R_1 = V_1$ . . . . . . $= \frac{7}{16}wl$

$R_2 = V_2 + V_3$ . . . . . $= \frac{5}{8}wl$

$R_3 = V_3$ . . . . . . $= -\frac{1}{16}wl$

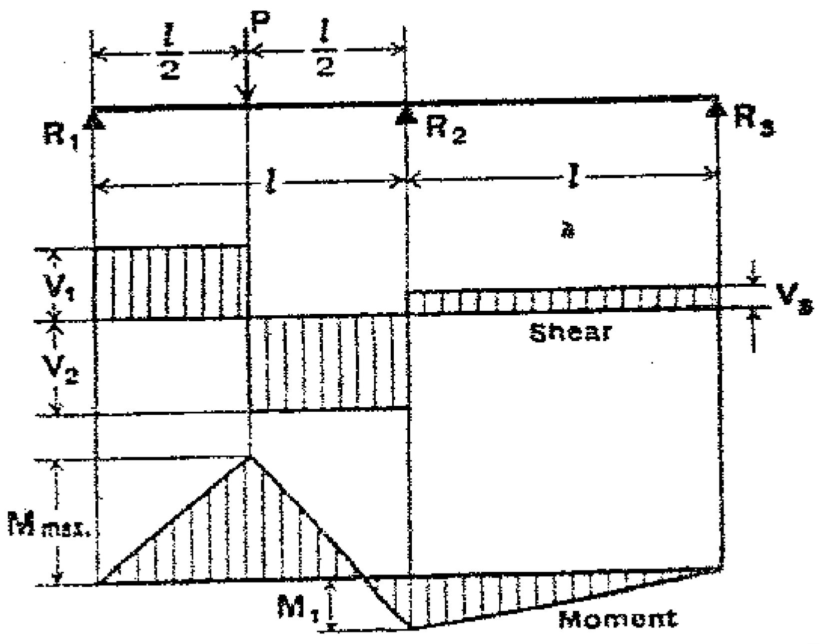
V_2 . . . . . . $= \frac{9}{16}wl$

M Max. (at $x = \frac{7}{16}l$) . . . $= \frac{49}{512}wl^2$

M_1 (at support R_2) . . . $= \frac{1}{16}wl^2$

M_x (when $x < l$) . . . $= \frac{wx}{16}(7l - 8x)$

30. CONTINUOUS BEAM—TWO EQUAL SPANS—CONCENTRATED LOAD AT CENTER OF ONE SPAN



Equivalent Tabular Load . $= \frac{13}{8}P$

$R_1 = V_1$ . . . . . . $= \frac{13}{32}P$

$R_2 = V_2 + V_3$ . . . . . $= \frac{11}{16}P$

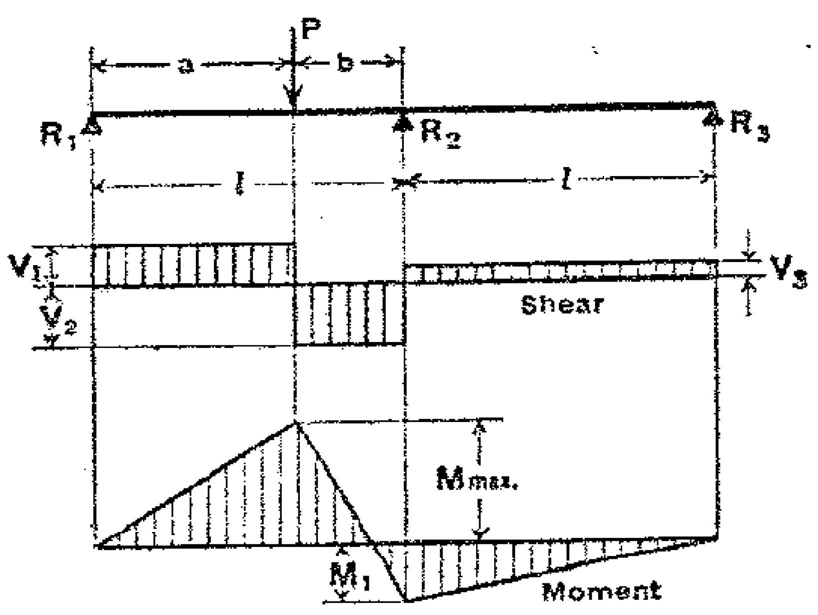
$R_3 = V_3$ . . . . . . $= -\frac{3}{32}P$

V_2 . . . . . . $= \frac{19}{32}P$

M Max. (at point of load) . $= \frac{13}{64}Pl$

M_1 (at support R_2) . $= \frac{3}{32}Pl$

31. CONTINUOUS BEAM—TWO EQUAL SPANS—CONCENTRATED LOAD AT ANY POINT



$R_1 = V_1$ . . . . . . $= \frac{Pb}{4l^3}\left(4l^2 - a(l+a)\right)$

$R_2 = V_2 + V_3$ . . . . . $= \frac{Pa}{2l^3}\left(2l^2 + b(l+a)\right)$

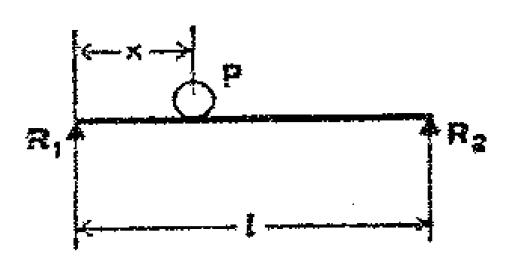
$R_3 = V_3$ . . . . . . $= -\frac{Pab}{4l^3}(l+a)$

V_2 . . . . . . $= \frac{Pa}{4l^3}\left(4l^2 + b(l+a)\right)$

M max. (at point of load) . $= \frac{Pab}{4l^3}\left(4l^2 - a(l+a)\right)$

M_1 (at support R_2) . $= \frac{Pab}{4l^2}(l+a)$

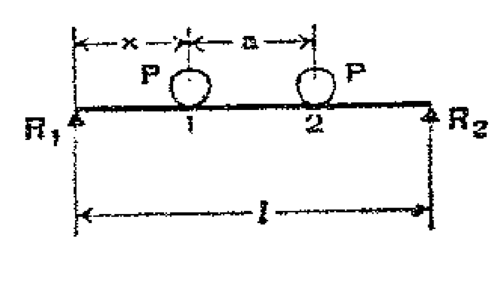
32. SIMPLE BEAM—ONE CONCENTRATED MOVING LOAD



R_1 max. $= V_1$ max. (at $x = 0$) . . . . $= P$

M max. (at point of load, when $x = \frac{l}{2}$) . $= \frac{Pl}{4}$

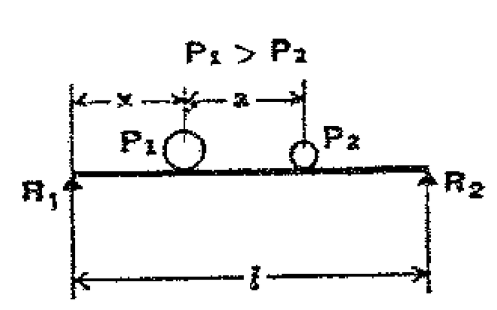
33. SIMPLE BEAM—TWO EQUAL CONCENTRATED MOVING LOADS



R_1 max. $= V_1$ max. (at $x = 0$) . . . . $= P\left(2 - \frac{a}{l}\right)$

M max.
- [when $a < (2 - \sqrt{2})\,l = .586l$ under load 1 at $x = \frac{1}{2}\left(l - \frac{a}{2}\right)$] $= \frac{P}{2l}\left(l - \frac{a}{2}\right)^2$
- [when $a > (2 - \sqrt{2})\,l = .586l$ with one load at center of span (case 32)] $= \frac{Pl}{4}$

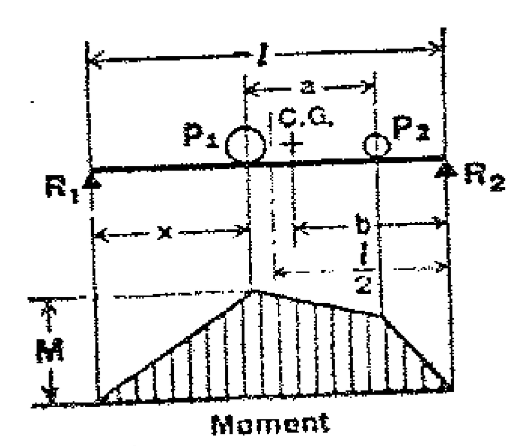
34. SIMPLE BEAM—TWO UNEQUAL CONCENTRATED MOVING LOADS



R_1 max. $= V_1$ max. (at $x = 0$) . . . . $= P_1 + P_2\frac{l-a}{l}$

M max.
- [under P_1, at $x = \frac{1}{2}\left(l - \frac{P_2a}{P_1 + P_2}\right)$] $= (P_1 + P_2)\frac{x^2}{l}$
- [M max. may occur with larger load at center of span and other load off span (case 32)] $= \frac{P_1l}{4}$

GENERAL RULES FOR SIMPLE BEAMS CARRYING MOVING CONCENTRATED LOADS



The maximum shear due to moving concentrated loads occurs at one support when one of the loads is at that support. With several moving loads, the location that will produce maximum shear must be determined by trial.

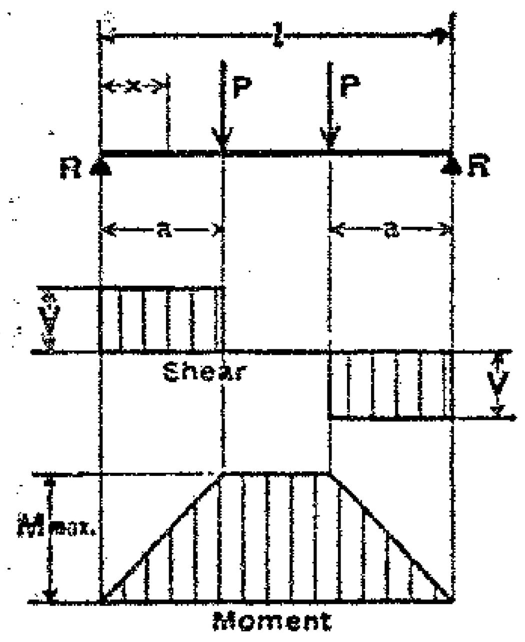
The maximum bending moment produced by moving concentrated loads occurs under one of the loads when that load is as far from one support as the center of gravity of all the moving loads on the beam is from the other support.

In the accompanying diagram, the maximum bending moment occurs under load P_1 when $x = b$. It should also be noted that this condition occurs when the center line of the span is midway between the center of gravity of loads and the nearest concentrated load.

From A.I.S.C. Steel Construction Manual - 1950.

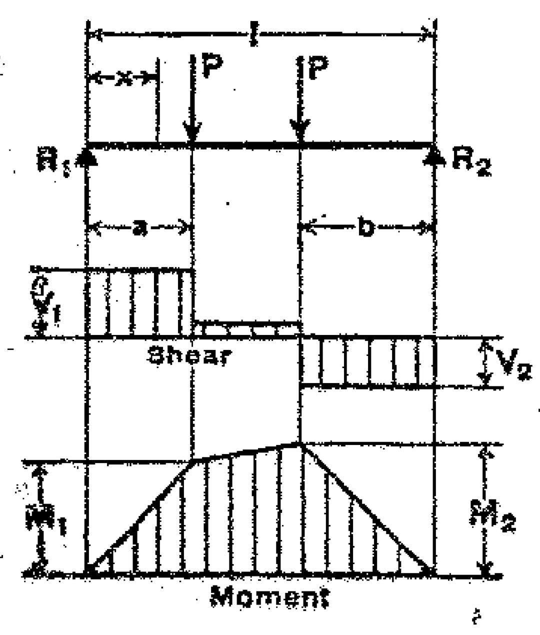
For nomenclature see page 8-01

9. SIMPLE BEAM—TWO EQUAL CONCENTRATED LOADS SYMMETRICALLY PLACED



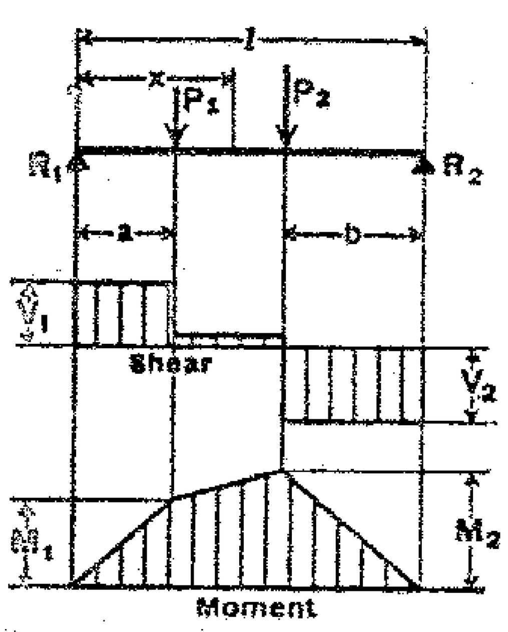
Equivalent Tabular Load	$= \frac{8Pa}{l}$
$R = V$	$= P$
M max. (between loads)	$= Pa$
M_x (when $x < a$)	$= Px$
Δmax. (at center)	$= \frac{Pa}{24EI}(3l^2 - 4a^2)$
Δ_x (when $x < a$)	$= \frac{Px}{6EI}(3la - 3a^2 - x^2)$
Δ_x (when $x > a$ and $< (l-a)$)	$= \frac{Pa}{6EI}(3lx - 3x^2 - a^2)$

10. SIMPLE BEAM—TWO EQUAL CONCENTRATED LOADS UNSYMMETRICALLY PLACED



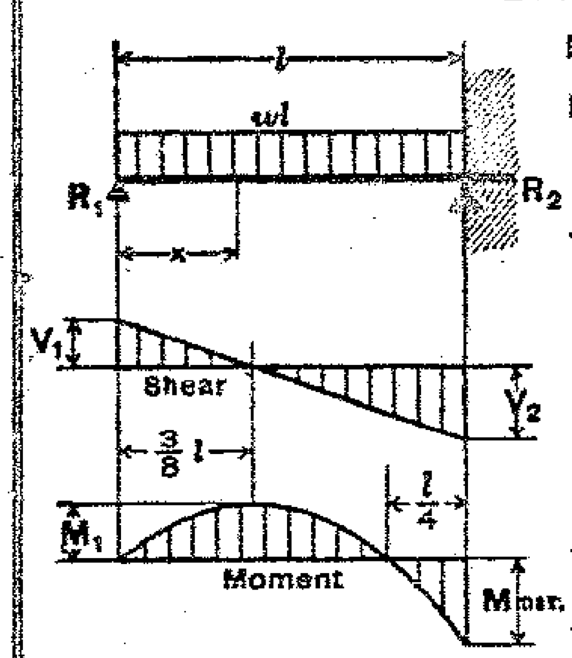
$R_1 = V_1$ (max. when $a < b$)	$= \frac{P}{l}(l - a + b)$
$R_2 = V_2$ (max. when $a > b$)	$= \frac{P}{l}(l - b + a)$
V_x (when $x > a$ and $< (l-b)$)	$= \frac{P}{l}(b - a)$
M_1 (max. when $a > b$)	$= R_1 a$
M_2 (max. when $a < b$)	$= R_2 b$
M_x (when $x < a$)	$= R_1 x$
M_x (when $x > a$ and $< (l-b)$)	$= R_1 x - P(x - a)$

11. SIMPLE BEAM—TWO UNEQUAL CONCENTRATED LOADS UNSYMMETRICALLY PLACED



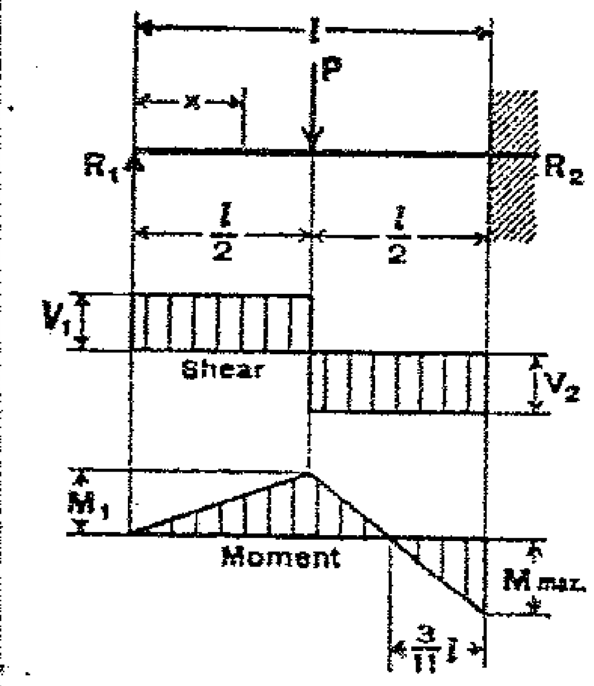
$R_1 = V_1$	$= \frac{P_1(l-a) + P_2 b}{l}$
$R_2 = V_2$	$= \frac{P_1 a + P_2(l-b)}{l}$
V_x (when $x > a$ and $< (l-b)$)	$= R_1 - P_1$
M_1 (max. when $R_1 < P_1$)	$= R_1 a$
M_2 (max. when $R_2 < P_2$)	$= R_2 b$
M_x (when $x < a$)	$= R_1 x$
M_x (when $x > a$ and $< (l-b)$)	$= R_1 x - P_1(x - a)$

12. BEAM FIXED AT ONE END, SUPPORTED AT OTHER—UNIFORMLY DISTRIBUTED LOAD



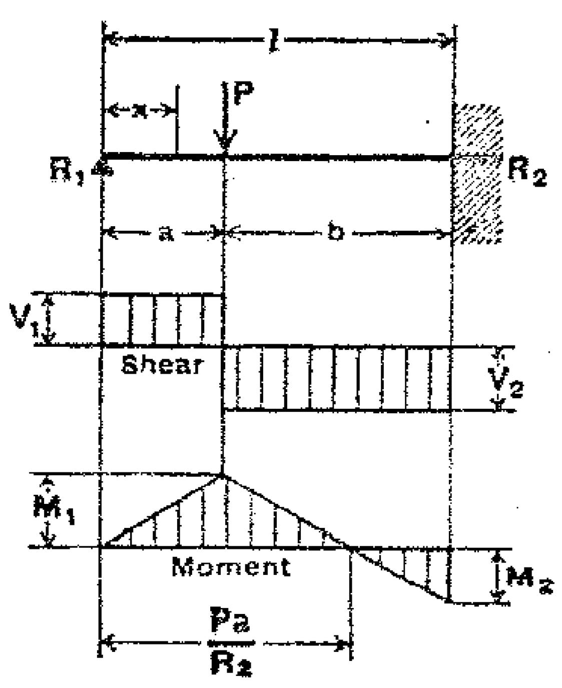
Equivalent Tabular Load	$= wl$
$R_1 = V_1$	$= \frac{3wl}{8}$
$R_2 = V_2$ max.	$= \frac{5wl}{8}$
V_x	$= R_1 - wx$
M max.	$= \frac{wl^2}{8}$
M_1 (at $x = \frac{3}{8}l$)	$= \frac{9}{128}wl^2$
M_x	$= R_1 x - \frac{wx^2}{2}$
Δmax. (at $x = \frac{l}{16}(1 + \sqrt{33}) = .4215l$)	$= \frac{wl^4}{185EI}$
Δ_x	$= \frac{wx}{48EI}(l^3 - 3lx^2 + 2x^3)$

13. BEAM FIXED AT ONE END, SUPPORTED AT OTHER—CONCENTRATED LOAD AT CENTER



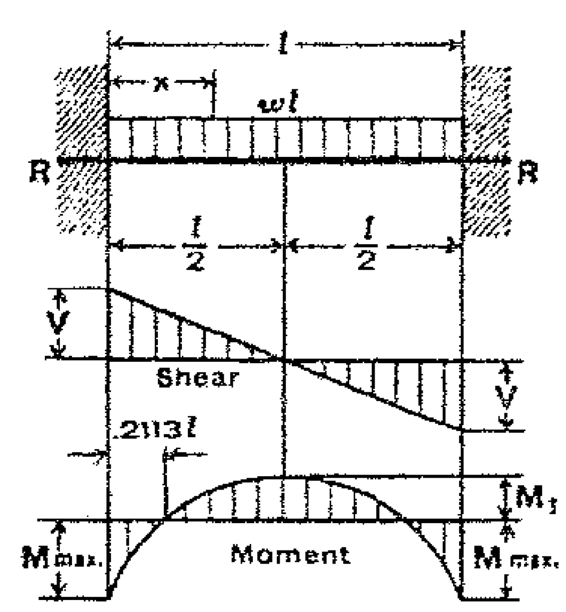
Equivalent Tabular Load	$= \frac{3P}{2}$
$R_1 = V_1$	$= \frac{5P}{16}$
$R_2 = V_2$ max.	$= \frac{11P}{16}$
M max. (at fixed end)	$= \frac{3Pl}{16}$
M_1 (at point of load)	$= \frac{5Pl}{32}$
M_x (when $x < \frac{l}{2}$)	$= \frac{5Px}{16}$
M_x (when $x > \frac{l}{2}$)	$= P\left(\frac{l}{2} - \frac{11x}{16}\right)$
Δmax. (at $x = l\sqrt{\frac{1}{5}} = .4472l$)	$= \frac{Pl^3}{48EI\sqrt{5}} = .009317\frac{Pl^3}{EI}$
Δ_x (at point of load)	$= \frac{7Pl^3}{768EI}$
Δ_x (when $x < \frac{l}{2}$)	$= \frac{Px}{96EI}(3l^2 - 5x^2)$
Δ_x (when $x > \frac{l}{2}$)	$= \frac{P}{96EI}(x - l)^2(11x - 2l)$

14. BEAM FIXED AT ONE END, SUPPORTED AT OTHER—CONCENTRATED LOAD AT ANY POINT



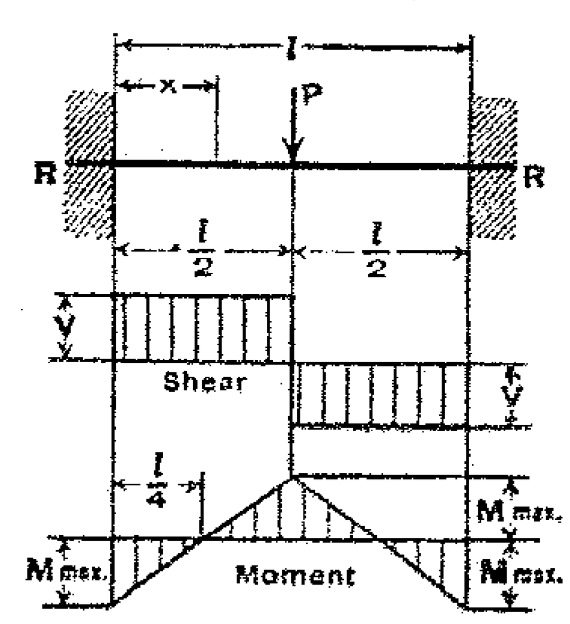
$R_1 = V_1$	$= \frac{Pb^2}{2l^3}(a + 2l)$
$R_2 = V_2$	$= \frac{Pa}{2l^3}(3l^2 - a^2)$
M_1 (at point of load)	$= R_1 a$
M_2 (at fixed end)	$= \frac{Pab}{2l^2}(a + l)$
M_x (when $x < a$)	$= R_1 x$
M_x (when $x > a$)	$= R_1 x - P(x - a)$
Δmax. (when $a < .414l$ at $x = l\frac{l^2 + a^2}{3l^2 - a^2}$)	$= \frac{Pa}{3EI}\frac{(l^2 - a^2)^3}{(3l^2 - a^2)^2}$
Δmax. (when $a > .414l$ at $x = l\sqrt{\frac{a}{2l + a}}$)	$= \frac{Pab^2}{6EI}\sqrt{\frac{a}{2l + a}}$
Δa (at point of load)	$= \frac{Pa^2b^3}{12EIl^3}(3l + a)$
Δ_x (when $x < a$)	$= \frac{Pb^2x}{12EIl^3}(3al^2 - 2lx^2 - ax^2)$
Δ_x (when $x > a$)	$= \frac{Pa}{12EIl^3}(l - x)^2(3l^2x - a^2x - 2a^2l)$

15. BEAM FIXED AT BOTH ENDS—UNIFORMLY DISTRIBUTED LOADS



Equivalent Tabular Load	$= \frac{2wl}{3}$
$R = V$	$= \frac{wl}{2}$
V_x	$= w\left(\frac{l}{2} - x\right)$
M max. (at ends)	$= \frac{wl^2}{12}$
M_1 (at center)	$= \frac{wl^2}{24}$
M_x	$= \frac{w}{12}(6lx - l^2 - 6x^2)$
Δmax. (at center)	$= \frac{wl^4}{384EI}$
Δ_x	$= \frac{wx^2}{24EI}(l - x)^2$

16. BEAM FIXED AT BOTH ENDS—CONCENTRATED LOAD AT CENTER



Equivalent Tabular Load	$= P$
$R = V$	$= \frac{P}{2}$
M max. (at center and ends)	$= \frac{Pl}{8}$
M_x (when $x < \frac{l}{2}$)	$= \frac{P}{8}(4x - l)$
Δmax. (at center)	$= \frac{Pl^3}{192EI}$
Δ_x	$= \frac{Px^2}{48EI}(3l - 4x)$

From A.I.S.C. Steel Construction Manual - 1950.